A Novel General Approximate Method for Lateral Load Analysis of Multistorey Building Frames

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Abstract: A general approximate method is propounded for the lateral load analysis of plane moment resisting multistory building frames. Since the approximate methods do not require properties of member cross section and materials, they are preferred for use in preliminary analysis. The versatility of the general method proposed is such that depending upon the load index chosen the method can be used for the analysis of short, medium and tall frames.

Keywords: Analysis, Approximate, Lateral load, Multistorey, Cantilever, Portal

Introduction:

Tall building construction is on the rise in the entire metropolis. These buildings are subjected to effects of wind and earthquake forces due to increased height. Owing to considerable difficulty in assessing the impacts of these loads in the building, approximate methods are resorted to in the beginning of the design process. Another advantage is that these methods do not require information on geometrical and material properties of the various members constituting the frame. Multistorey building design is an iterative procedure. Several alternatives have to be examined for evaluating best member dimensions. For arriving at the optimal member sizes, judicious choice of a method in preliminary analysis curtails the number of cycles facilitating easy reach of the final solution in one or two repetitions. Regular moment resisting framed building can be analysed as a plane frame building even though modern computers have the capability to perform three dimensional analyses. However, the restriction of computer use is that member properties (b, d, or I) and material properties i.e., Young’s modulus (E), Transverse modulus (G) and Poisson’s ratio (□) are necessary for use as input. Experienced analyst and architects will be in a position to predict ‘a priori’ member dimensions for the beams and columns. However, their estimation will be subjected to variation from time to time and may differ from person to person. In general, such empirical decisions may not be consistent. Hence, if a sound approximate method is used during early stages; personal errors will not creep in to the solution. To overcome these difficulties, preliminary analysis is adopted using the approximate methods. These approximate methods are based on some assumptions (references 1 and 2). The procedures are independent of geometrical and material properties. This unique feature makes them attractive for getting reasonable values for the stress resultants in the beginning. Utility of the approximate methods are given in reference 3.

Need for the Study:

For the analysis of short frames currently portal method is used and for long frames cantilever method is used. The flaw of the portal method is that it predicts the same magnitude for terminal moments. The cantilever method is tedious to work with. The guidelines for deciding the frames as short or long are indistinct. Hence a general method for lateral load analysis of short, medium and tall moment resisting frames is of significance.

Characteristics of short, Intermediate (medium) and tall Moment Resistant Frames:

a. Short frame: In short frames, the panel distortion (shear mode) is preponderantly due to lateral sway with
negligible bending as shown in Figure 1(a). Since the flexural deformation is almost absent, the inner columns do not suffer any axial compression or elongation. Hence, without introducing much error, it is assumed that the axial forces in interior columns are zero. For analyzing short frames, currently the portal method is used. The flaw of the method is that it predicts the same magnitude for beam terminal moments in all the bays of the storey which is contrary to engineering expectations. Consequence of this assumption is that axial forces of moderate magnitude are produced in the inner columns of the storey which is incorrect. However, the forte of the method is its simplicity. It is stated that the portal method can be used for frames with height to width ratio less than five or for frames with number of storeys less than twenty five.

b. Medium rise frame: In these frames both shear and bending deformations are effective. Bending behaviour is beginning to manifest as the height increases. At present no method is available for classifying this type of frame. Either the portal or cantilever method is used depending upon the discretion of the analyst.

c. Tall frame: In tall frames, the flexural mode of distortion is outweighing and the shearing phenomenon is negligible. The flexural behaviour of a tall frame is shown in Figure 1(b). For the analysis of this frame, the cantilever method is recommended. The potency of the method is such that it has withstood the test of time and onslaught of the computer oriented procedures. Because of its powerful concept, some engineers prefer to use the solution of this method as the final result. The slight blemish of the cantilever method is that it is somewhat tedious to work with. The method is useful for frames with height to width ratio greater than five or frames with 25 to 35 storeys. Currently, the guidelines available in the literature for deciding a frame as short or tall are indistinct. Therefore, the analyst can use his ability to make a reasonable decision, whether the frame is short, medium or tall.

**Proposed Analysis:**

To begin with, it is assumed that hinges are located in the middle of all columns and beams. Figure 2 shows the top most floor above the hinges in the columns of a multistorey building frame consisting of two bays with width \( l_1 \) and \( l_2 \). The total length of the bays is \( L = l_1 + l_2 \). The storey height is 'h'. Since the hinges are placed in the middle of all the columns, the column terminal moments are found as \( V_i \times 0.5h \), where \( V_i \) is the shear in the column, i. Having found the various column moments using the information provided subsequently, the beam terminal moments are determined.
**Determination of $V_i$:**

The nodal load $P$ acting at top left corner of the frame shown in Fig.2 can be distributed equally as $0.5P$ at either end of the beam of the floor, using the principle of transmissibility. The loading at the two ends is distributed along the length of the beam as uniformly varying load whose distribution is shown in Figure 3. It consists of a rectangle and a parabola. Now, the storey which consists of two bays is decomposed into two aisles carrying nodal load $T_1$ and $T_2$. The subscripts show the number of the bay. Generally, it is denoted as $T_n$. Each column carries $0.5T_1$ and $0.5T_2$ as shown in Figure 4. The nodal loads are given by

$$T_1 = \int_{0}^{l_1} f(x) \, dx \quad (1)$$

$$T_2 = \int_{l_1}^{L} f(x) \, dx \quad (2)$$

In order to retrieve the original structure, the two decomposed aisles are assembled back.

Then the shears $V_i$ of each column in the topmost floor of the frame shown in Figure 2 are as given below.

$$V_1 = 0.5T_1 \quad (3)$$

$$V_2 = 0.5(T_1 + T_2) \quad (4)$$

$$V_3 = 0.5T_2 \quad (5)$$

**Proposed Hypothesis for Load Distribution:**

A tentative assumption is made for the load distribution in order to draw out and test its logical consequences. The distribution of the load is shown in Figure 3. The storey shear $P$ is distributed between the rectangle and the parabola. For this purpose a parameter known as “load index” denoted as $RXP$ is made use of. $RXP$ means the rectangular portion carries X percent of total storey shear $P$. For example, $R75P$ indicates, the rectangular section carries 75% of total storey shear $P$ and 25% carries by the parabola. In the present study, three levels of load percentages are considered. They are $R100P$, $R75P$ and $R50P$. 
**Figure 2:** Topmost Floor above the Hinges

**Figure 3:** Distribution of Loading on Beam in Topmost Floor
Before proceeding to an illustrative example, the following information will be useful for expeditiously performing the analysis.

1. The storey number is counted from the top most floor. The small letter “j” is used for this purpose.
2. The bays are counted from left and “n” is used as suffix to indicate the particular bay.
3. All the column terminal moments will be in the clockwise direction and beam moments will be in the anticlockwise direction.
4. In any joint, sum of column moments=sum of beam moments.
5. In any member (beam or column), when the hinge is located in the middle, the moments in the two ends will be same in magnitude and direction.
6. To begin with, the analysis should start from the top most floor and then proceeded in the downward direction. Because of this procedure, the numbering of the storeys are started from the top most storey.
7. For computing the beam moments in any floor, the process is initiated from the left most joint and then proceeded towards right in conjunction with information number 5.
8. The nodal load $T_n=T_{nq}+T_{nr}$

Where $T_{nq}$=nodal load due to parabolic distribution in aisle number ‘n’

$T_{nr}$=nodal load due to rectangular distribution in aisle number ‘n’.

**Illustrative Example:**

This example is from reference 4. All the data are furnished in Figure 5. The frame consists of three bays. Computation of nodal load T for the second bay will be illustrated. Hence n=2.
Then $T_2 = T_{2q} + T_{2r}$
The load index considered is R50P
L= $l_1 + l_2 + l_3 = 20 + 25 + 30 = 75$ m
$l_n = 25$ m

(a) **Topmost floor (j=1, n=2):**
The maximum ordinate ‘q’ of the parabola is as follows:
\[
\frac{2}{3} q L = 0.5 P
\]
The storey shear, $P=10$ kN
Hence $q = \frac{15}{150}$ kN

For the rectangle, $t x L = 0.5 P$
\[
\therefore \ t = \frac{15}{150} \text{ kN/m run}
\]
\[
T_{2r} = \int_{0}^{L} t dx = 1.666 \text{ kN}
\]
\[
T_{2q} = \int_{0}^{L} \frac{45}{20} q(L-x)^2 \frac{dx}{L^2} = 2.363 \text{ kN}
\]
They are
\[
T_1 = 2.2103 \text{ kN}
\]
\[
T_3 = 3.7600 \text{ kN}
\]
The column shears are
\[
V_1 = 0.5 T_1 = 1.105 \text{ kN}
\]
\[
V_2 = 0.5(T_1 + T_3) = 3.119 \text{ kN}
\]
\[
V_3 = 0.5(T_2 + T_3) = 3.895 \text{ kN}
\]
\[
V_4 = 0.5 T_3 = 1.880 \text{ kN}
\]
Check $\sum V = 9.999 \text{ kN} = 10 \text{ kN}$, O.K.
For this storey, $h=15$ m. The column terminal moments are found using $V_i \times 0.5h$.
The details are given in Table 1 along with exact results.

(b) **Second floor (j=2, n=2):**
The only difference lies in the value of the storey shear $P$ which is 10+10=20 kN
h = 20 m. With this data
\[
V_1 = 2.210 \text{ kN}
\]
\[
V_2 = 6.238 \text{ kN}
\]
\[
V_3 = 7.790 \text{ kN}
\]
\[
V_4 = 3.760 \text{ kN}
\]
Table 1: Prediction by the General Method and Comparison

<table>
<thead>
<tr>
<th>No.</th>
<th>Member (columns)</th>
<th>Portal Method kNm</th>
<th>R100P kNm</th>
<th>R75P kNm</th>
<th>R50P kNm</th>
<th>Cantilever Method kNm</th>
<th>Exact Method kNm</th>
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Table 2: Prediction by the General Method and Comparison

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The column moments ($V \times 0.5h$) were computed and are shown in Table 1. Having computed column terminal moments in this manner, using information no. 4, 5 and 7, the beam moments were found and are tabulated in Table 2 along with exact results. In the manner discussed above, the moments were calculated for R75P and R100P loading cases and tabulated in Table 1 and Table 2 along with the solutions of the portal and cantilever methods.

Discussions:
Case (a) R100P: It is seen in Table 1 and Table 2 that for this loading, the beam terminal moments in all the bays are different in magnitude unlike the portal method. Again, it is found that the axial forces in the inner columns vanish. Thus, the two flaws of portal method are removed. The implication is that the loading index R100P is valid for short frame. In other words, the distribution of the nodal loads is purely a rectangular one.
Case (b) R50P: It is seen from the Table 1 and Table 2 that the solution for this load index coincides almost with that of the cantilever method. The inference is the frame is a flexural frame and hence can be classified as a tall frame.

Case (c) R75P: Most of the results of this load index lie in between the short and tall frame. Therefore, R75P can be made use of for analyzing intermediate (medium rise) frames.

Summarizing, the following deduction is arrived at

R100P                          Short frame  
R75P                            Medium rise frame  
R50P                            Tall frame

Recapitulation:
A general method is set forth for the lateral load analysis of multistorey building frames. By suitable choice of the loading index, the method can be used for the analysis of short, medium and tall frames. In the proposed method, the flaws of the portal method are eliminated. For tall frames, the results agree well with that of the cantilever method. In fine, it is stated that the method advanced in this paper has the capability to accommodate all the three kinds of frames depending upon the load index chosen in the distribution function, i.e., R100P, R75P, and R50P.

References: